15.12.2023

1) $\int_0^1 \arctan(x-1)\,dx$; per parti con

$f'(x)=1 \Rightarrow f(x)=x$

$g(x)=\arctan(x-1)$

$g'(x)=\frac{1}{1+(x-1)^2}$

$$\int_0^1 \arctan(x-1)\,dx = \underbrace{\left[x\arctan(x-1)\right]_0^1}_{0} - \int_0^1 \frac{x}{1+(x-1)^2}\,dx$$

$$= -\frac{1}{2}\int_0^1 \frac{2(x-1)}{1+(x-1)^2}\,dx - \int_0^1 \frac{1}{1+(x-1)^2}\,dx$$

$$= -\frac{1}{2}\left[\log\left[1+(x-1)^2\right]\right]_0^1 - \left[\arctan(x-1)\right]_0^1$$

$$= +\frac{1}{2}\log 2 + \underbrace{\arctan(-1)}_{-\frac{\pi}{4}} = \frac{1}{2}\log 2 - \frac{\pi}{4}$$

2) 10.2.2023

$$\int_0^{\frac{\pi}{2}} \frac{(\cos x - 1)\sin x}{\cos^2 x + 4\cos x + 5}\,dx$$

$\sin x\,dx = -dt$

dispari in $\sin x$ $\Rightarrow$ Sost.: $t=\cos x$

$\Rightarrow dt = -\sin x\,dx$

$$= -\int_1^0 \frac{t-1}{t^2+4t+5}\,dt = \int_0^1 \frac{t-1}{t^2+4t+5}\,dt$$

scrivo $t^2+4t+5 = (t+2)^2+1$

e
$$\frac{t-1}{1+(t+2)^2} = \frac{1}{2}\,\frac{2(t+2)}{1+(2+t)^2} - \frac{3}{1+(t+1)^2}$$

Quindi

$$\int_0^1 \frac{t-1}{1+(t+2)^2}\,dt = \frac{1}{2}\int_0^1 \frac{2(t+2)}{1+(t+2)^2}\,dt - 3\int_0^1 \frac{1}{1+(t+2)^2}\,dt$$

$$= \frac{1}{2}\Big[\log\big(1+(t+2)^2\big)\Big]_0^1 - 3\Big[\arctan(t+2)\Big]_0^1$$

$$= \frac{1}{2}\left(\log 10 - \log 5\right) - 3\arctan(3) + 3\arctan(2)$$

$$= \frac{1}{2}\log(2) - 3\arctan(3) + 3\arctan(2)$$

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3) 3.4.2023

$$\int_0^1 x\log(1+x^2)\,dx \quad ; \quad \text{per parti con}$$

$$f'(x) = x \Rightarrow f(x) = \frac{x^2}{2}$$

$$g(x) = \log(1+x^2) \Rightarrow g'(x) = \frac{2x}{1+x^2}$$

$$\int_0^1 x\log(1+x^2)\,dx = \underbrace{\left[\frac{x^2}{2}\log(1+x^2)\right]_0^1}_{=\ \frac{1}{2}\log 2} - \frac{1}{2}\int_0^1 \underbrace{x^2\,\frac{2x}{1+x^2}}_{=\ 1-\frac{1}{1+x^2}}\,dx$$

$$= \frac{1}{2}\log 2 - \frac{1}{2}\int_0^1 \left(1 - \frac{1}{1+x^2}\right) 2x \, dx$$

$$= \frac{1}{2}\log 2 - \underbrace{\int_0^1 x \, dx}_{1/2} + \frac{1}{2}\int_0^1 \frac{2x}{1+x^2} \, dx$$

$$= \frac{1}{2}\log - \frac{1}{2} + \frac{1}{2}\underbrace{\left[\log(1+x^2)\right]_0^1}_{\log 2}$$

$$= \log 2 - \frac{1}{2} \qquad (>0)$$

$$\frac{x^2}{1+x^2} = 1 - \frac{1}{1+x^2}$$

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4) 21.6. 2023

$$\int_{e^{-1}}^1 \frac{\log x + 2}{(\log x)^2 + 2\log x + 2} \frac{dx}{x}$$

Sost.: $t = \log x$

$dt = \frac{dx}{x}$

$$= \int_{-1}^0 \frac{t+2}{t^2 + 2t + 2} \, dt = \int_{-1}^0 \frac{t+2}{1+(t+1)^2} \, dt$$

$$= \frac{1}{2}\int_{-1}^0 \frac{2(t+1)}{1+(t+1)^2} + \int_{-1}^0 \frac{1}{1+(t+1)^2} \, dt$$

$$= \frac{1}{2}\left[\log(1+(t+1)^2)\right]_{-1}^0 + \left[\arctan(t+1)\right]_{-1}^0$$

$$= \frac{1}{2}\log 2 + \frac{\pi}{4}$$

5) 12.7. 2023

$$\int_0^{\pi^2} \sin(\sqrt{x})\,dx \, ;$$

Sost.: $t = \sqrt{x}$

$$dt = \frac{dx}{2\sqrt{x}} \Rightarrow dx = 2\sqrt{x}\,dt = 2t\,dt$$

Quindi

$$\int_0^{\pi^2} \sin(\sqrt{x})\,dx = 2\int_0^{\pi} t \sin t\,dt$$

per parti con

$$f'(t) = \sin t \Rightarrow f(t) = -\cos t$$

$$g(t) = t \Rightarrow g'(t) = 1$$

$$\Rightarrow 2\int_0^{\pi} t \sin t\,dt = \Big[-2t\cos t\Big]_0^{\pi} + 2\int_0^{\pi} \cos t\,dt$$

$$= 2\pi + \underbrace{2\Big[\sin t\Big]_0^{\pi}}_{=0} = \underline{2\pi}$$

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6) <u>4. 9. 2023</u>:

$$\int_0^1 \frac{\sqrt{x}}{1+(\sqrt{x}+1)^2}\,dx$$

Sost.: $t = \sqrt{x} + 1$

$$dt = \frac{dx}{2\sqrt{x}}$$

$$\Rightarrow dx = 2\sqrt{x}\,dt = 2(t-1)\,dt$$

Quindi

$$\int_0^1 \frac{\sqrt{x}}{1+(\sqrt{x}+1)^2}\,dx = \int_1^2 \frac{2(t-1)^2}{1+t^2}\,dt$$

$$= 2\int_1^2 \frac{t^2-2t+1}{1+t^2}\,dt = 2\int_1^2 \left(1 - \frac{2t}{1+t^2}\right) dt$$

$$= 2\left(1 - \left[\log(1+t^2)\right]_1^2\right)$$

$$= 2 - 2\log\left(\frac{5}{2}\right) \qquad (>0)$$

---

7) 14. 1. 2019

$$I = \int_0^4 e^{\sqrt{x}}\,dx \; ; \qquad \text{Sost.: } t = \sqrt{x}$$

$$dt = \frac{dx}{2\sqrt{x}} \Rightarrow dx = 2t\,dt$$

Quindi

$$I = \int_0^2 2t\,e^t\,dt \; ; \qquad \text{per parti:}$$

$$2t = g(t) \Rightarrow g'(t) = 2$$

$$f'(t) = e^t \Rightarrow f(t) = e^t$$

$$I = \underbrace{\left[2te^t\right]_0^2}_{4e^2} - 2\int_0^2 e^t\,dt$$

$$= 4e^2 - 2\left[e^t\right]_0^2 = 2e^2 + 2$$

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8) 4. 2. 2019:

$$I = \int_0^{\pi/2} \frac{\cos x + \sin(2x)}{1 + \sin^2 x}\,dx \; ; \qquad \sin(2x) = 2\sin x\cos x$$

$$= \int_0^{\frac{\pi}{2}} \underbrace{\frac{\cos x + 2\sin x\cos x}{1+\sin^2 x}}_{\text{dispari in } \cos x}\,dx \Rightarrow t = \sin x$$

$$dt = \cos x\,dx$$

$$= \int_0^1 \frac{1+2t}{1+t^2}\,dt$$

$$= \int_0^1 \frac{1}{1+t^2}\,dt + \int_0^1 \frac{2t}{1+t^2}\,dt$$

$$= \Big[\arctan(t)\Big]_0^1 + \Big[\log(1+t^2)\Big]_0^1 = \frac{\pi}{4} + \log 2$$

9) 16.4.2019

$$I = \int_0^1 \frac{\sqrt{x}}{2+\sqrt{x}}\,dx \quad ; \quad \text{Sost: } t = \sqrt{x} + 2$$

$$dt = \frac{dx}{2\sqrt{x}}$$

Quindi $\Rightarrow dx = 2\sqrt{x}\,dt = 2(t-2)\,dt$

$$I = \int_2^3 \frac{(t-2)\;2(t-2)}{t}\,dt$$

$$= 2\int_2^3 \frac{t^2-4t+4}{t}\,dt = \int_2^3 \left(2t - 8 + \frac{8}{t}\right)dt =$$

$$= \Big[t^2\Big]_2^3 - \Big[8t\Big]_2^3 + 8\Big[\log t\Big]_2^3$$

$$= 5 - 8 + 8\log 3/2 = 8\log(3/2) - 3$$

10) 2.7.2019

$$\int_0^{\log 2} \frac{2e^x\,dx}{e^{2x}+2e^x+1} \qquad \text{Sost: } t = e^x$$

dt

$$dt = e^x\,dx$$

$$= \int_1^2 \frac{2}{t^2+2t+1}\,dt = 2\int_1^2 \frac{1}{(t+1)^2}\,dt$$

$$= 2\left[-\frac{1}{t+1}\right]_1^2$$

$$= 2\left[-\frac{1}{3}+\frac{1}{2}\right] = \frac{1}{3}$$ E

---

11) 30.8.2019

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos^3 x + 2x)\, dx$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2x\, dx = \left[x^2\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 0$$

Più in generale: se f è dispari, allora

$\forall a > 0$: $\displaystyle\int_{-a}^{a} f(x)\, dx = 0$

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^3 x\, dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\,(1-\sin^2 x)\, dx$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\, dx - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \sin^2 x\, dx$$

$t = \sin x$

$dt = \cos x\, dx$

$$= \underbrace{\left[\sin x\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}}_{= 2} - \int_{-1}^{1} t^2\, dt$$

$$= 2 - \left[\frac{t^3}{3}\right]_{-1}^{1} = 2 - \frac{2}{3} = \frac{4}{3}$$ $\boxed{B}$

10.7.2018

$$I = \int_3^5 \frac{1}{(x-2)\sqrt{x-1}}\, dx$$

Sost.: $t = \sqrt{x-1}$, $\Rightarrow x = t^2 + 1$

$$\Rightarrow \quad dt = \frac{dx}{2\sqrt{x-1}} \Rightarrow \frac{dx}{\sqrt{x-1}} = 2\,dt$$

Quindi

$$I = 2\int_{\sqrt{2}}^{2} \frac{1}{t^2-1}\,dt$$

$$= 2\int_{\sqrt{2}}^{2} \frac{1}{(t-1)(t+1)}\,dt = \int_{\sqrt{2}}^{2} \left(\frac{1}{t-1} - \frac{1}{t+1}\right) dt$$

$$= \int_{\sqrt{2}}^{2} \frac{1}{t-1}\,dt - \int_{\sqrt{2}}^{2} \frac{1}{1+t}\,dt =$$

$$= \left[\log(t-1)\right]_{\sqrt{2}}^{2} - \left[\log(t+1)\right]_{\sqrt{2}}^{2}$$

$$= -\log(\sqrt{2}-1) - \left(\log 3 - \log(1+\sqrt{2})\right)$$

$$= \log\frac{1+\sqrt{2}}{\sqrt{2}-1} - \log 3 \qquad E$$

$$= \left(\log\frac{1}{3} - \log\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)$$

18.12.2023.

1) 13.1.2023:

$$f(x) = \begin{cases} \dfrac{(\sin x)^2 - \sin(x^2)}{x^\alpha} & \text{per } x > 0 \\ \arctan(x^2) & \text{per } x \le 0 \end{cases}$$

Per quali $\alpha > 0$ f risulta derivabile in $x_0 = 0$?

Derivata sinistra:

$$f'_-(0) = \lim_{x \to 0-} \frac{f(x) - \overbrace{f(0)}^{=0}}{x} = \lim_{x \to 0-} \frac{\arctan(x^2)}{x}$$

$$= \lim_{x \to 0-} \frac{x^2}{x} = 0$$

Derivata destra:

$$f'_+(0) = \lim_{x \to 0+} \frac{f(x) - \overbrace{f(0)}^{=0}}{x} = \lim_{x \to 0+} \frac{(\sin x)^2 - \sin(x^2)}{x^{1+\alpha}}$$

Taylor:
$$\sin x = x - \frac{x^3}{6} + o(x^3) \qquad x \to 0$$
$$\sin(x^2) = x^2 - \frac{x^6}{6} + o(x^6) \qquad x \to 0$$

$$\Rightarrow \quad (\sin x)^2 = \left(x - \frac{x^3}{6} + o(x^3)\right)^2 = x^2 - \frac{x^4}{3} + o(x^4)$$

Quindi
$$(\sin x)^2 - \sin(x^2) = -\frac{x^4}{3} + o(x^4)$$

$$\Rightarrow \quad f'_+(0) = 0 \iff \lim_{x \to 0+} \frac{-\frac{x^4}{3}}{x^{1+\alpha}} = -\frac{1}{3} \lim_{x \to 0+} x^{3-\alpha} = 0$$

$\iff \quad \alpha < 3$

Conclusione:

f è derivabile in $x_0 = 0$ se e solo se $\alpha < 3$.

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2) 10.2.2023:

$$f(x) = \begin{cases} \dfrac{\log(1+x^3) - \sin(x^3)}{x^\alpha} & \text{per } x > 0 \\ 1 - \cos(x^2) & \text{per } x \le 0 \end{cases}$$

---

f derivabile in $x_0 = 0$?

Derivata sinistra

$$f'_-(0) = \lim_{x \to 0^-} \frac{f(x) - \overbrace{f(0)}^{=0}}{x} = \lim_{x \to 0^-} \frac{1 - \cos(x^2)}{x} \overset{[H]}{=}$$

$$= \lim_{x \to 0^-} \frac{-2x \sin(x^2)}{1} = 0$$

Derivata destra: Taylor:

$$\log(1+t) = t - \frac{t^2}{2} + o(t^2) \quad t \to 0$$

$$\Rightarrow \quad \log(1+x^3) = x^3 - \frac{x^6}{2} + o(x^6) \quad x \to 0$$

$$\sin(x^3) = x^3 - \frac{x^9}{6} + o(x^9) \quad x \to 0$$

$$\Rightarrow \quad \log(1+x^3) - \sin(x^3) = -\frac{x^6}{2} + o(x^6)$$

Quindi $f'_+(0) = 0 \iff \lim_{x \to 0^+} \dfrac{-\frac{x^6}{2}}{x^{1+\alpha}} = 0$

$$\Leftrightarrow \quad \lim_{x\to 0+} x^{5-\alpha} = 0$$

$$\Leftrightarrow \quad \underline{\alpha < 5}.$$

<u>Conclusione</u>

f è derivabile in $x_0 = 0 \Leftrightarrow \alpha < 5$

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3) 21.6.2023

$$f(x) = \begin{cases} e^{-1}(x+1)^{\frac{2}{x}} & \text{per } x > 0 \\ 1 & \text{per } x = 0 \\ \exp\left[\arctan\left(\frac{\pi}{4|x|^{\alpha-1}}\right) + 1\right] & \text{per } x < 0 \end{cases}$$

Per quali $\alpha \in \mathbb{R}$ f ammette un p. di discontinuità eliminabile in $x = 0$?

<u>Limite destro</u>: $f_+(0) = \lim_{x\to 0+} f(x) = \lim_{x\to 0+} e^{-1}(x+1)^{\frac{2}{x}}$

$$= e^{-1} \lim_{x\to 0+} e^{\frac{2}{x}\log(1+x)} = e^{-1} e^{\lim_{x\to 0+} \frac{2\log(1+x)}{x}}$$

2

$$= e^{-1} e^{2} = \underline{e}$$

$\Rightarrow$ 0 è un p. di discontinuità eliminabile $\Leftrightarrow$

$$f_-(0) = \lim_{x\to 0-} f(x) = e$$

$$\Leftrightarrow \quad \lim_{x\to 0-} \left[\arctan\left(\frac{\pi}{4|x|^{\alpha-1}}\right) + 1\right] = 1$$

$$\Leftrightarrow \lim_{x\to 0-} \arctan\left(\frac{\pi}{4|x|^{\alpha-1}}\right) = 0$$

$$\Leftrightarrow \lim_{x\to 0-} \frac{\pi}{4|x|^{\alpha-1}} = 0 \Leftrightarrow \underline{\alpha < 1}$$

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4) 12.7.2023:

$$f(x) = \begin{cases} x(1+x+x^2)^{\frac{\alpha}{x}} & \text{per } x > 0 \\ \sin(x^2) + \sin(2x) & \text{per } x \leq 0 \end{cases}$$

Per quali $\alpha \in \mathbb{R}$ f risulta derivabile in $x = 0$?

<u>Derivata sinistra</u>:

$$f'_-(0) = \lim_{x\to 0-} \frac{f(x) - \overbrace{f(0)}^{=0}}{x} = \lim_{x\to 0-} \frac{\sin(x^2) + \sin(2x)}{x}$$

$$= \underbrace{\lim_{x\to 0-} \frac{\sin(x^2)}{x}}_{=0} + \underbrace{\lim_{x\to 0-} \frac{\sin(2x)}{x}}_{=2} = 2$$

<u>Derivata destra</u>

$$f'_+(0) = \lim_{x\to 0+} \frac{f(x)}{x} = \lim_{x\to 0+} (1+x+x^2)^{\frac{\alpha}{x}}$$

$$= \lim_{x\to 0+} e^{\frac{\alpha}{x}\log(1+x+x^2)}$$

$$= e^{\lim_{x\to 0+} \frac{\alpha \log(1+x+x^2)}{x}}$$

$$\lim_{x\to 0+} \frac{\log(1+x+x^2)}{x} \overset{H}{=} \lim_{x\to 0+} \frac{\frac{1+2x}{1+x+x^2}}{1} = 1$$

$$\Rightarrow \quad f'_+(0) = e^{\alpha}$$

Quindi f e' derivabile in $x=0$ se e solo se $e^{\alpha} = 2 \iff \underline{\alpha = \log(2)}$

20.1.2022 : $\displaystyle\sum_{n=1}^{\infty} \underbrace{\log\left(\cos^2\left(\tfrac{1}{n}\right)\right)\left(\sqrt{n^3+n^{2\beta}} - n^{\beta}\right)}_{= a_n}$

per quali $\beta \in \mathbb{R}$ la serie converge ?

$a_n \leq 0 \quad \forall \quad n \geq 1$

Comportamento asintotico di a_n :

$$\log\left(\cos^2\left(\tfrac{1}{n}\right)\right) = 2\log\left(\cos\left(\tfrac{1}{n}\right)\right)$$

Taylor : $\cos\left(\frac{1}{n}\right) = 1 - \frac{1}{2n^2} + o(n^{-2}) \quad n\to\infty$

perche' $\cos(x) = 1 - \frac{x^2}{2} + o(x^2) \quad x \to 0$

$$\Rightarrow \log\left(\cos\left(\tfrac{1}{n}\right)\right) = \log\left(1 - \frac{1}{2n^2} + o(n^{-2})\right)$$

$$\overset{(\text{Taylor})}{=} -\frac{1}{2n^2} + o(n^{-2}) \quad n\to\infty$$

$$\Rightarrow \log\left(\cos^2\left(\tfrac{1}{n}\right)\right) \sim -\frac{1}{n^2} \quad n\to\infty$$

$$\sqrt{n^3+n^{2\beta}} - n^\beta = \frac{n^3 + \cancel{n^{2\beta}} - \cancel{n^{2\beta}}}{\sqrt{n^3+n^{2\beta}} + n^\beta} = \frac{n^3}{\sqrt{n^3+n^{2\beta}} + n^\beta}$$

- se $\beta > \frac{3}{2}$, allora $\sqrt{n^3+n^{2\beta}} + n^\beta \sim 2n^\beta \quad n\to\infty$

- se $\beta \leq \frac{3}{2}$, allora $\sqrt{n^3+n^{2\beta}} + n^\beta \sim c\,n^{3/2} \quad n\to\infty$

Quindi

$$a_n \sim -\frac{1}{n^2}\,\frac{n^{3-\beta}}{2} \qquad \text{se} \quad \beta > 3/2$$

oppure

$$a_n \sim -\frac{1}{n^2}\,n^{3/2} \qquad \text{se} \quad \beta \leq 3/2$$

$\Rightarrow$ la serie non converge se $\beta \leq 3/2$ perché

in tal caso $a_n \sim -\frac{1}{\sqrt{n}}$ e

la serie $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverge

Se $\beta > 3/2$, allora

$$a_n \sim -\frac{n^{1-\beta}}{2} \qquad n\to\infty$$

$\Rightarrow$ per il criterio del confronto asintotico la serie converge se e solo se

$$1-\beta < -1 \iff \beta > 2$$

Ricordo: $\sum_{n=1}^{\infty} \frac{1}{n^\alpha}$ converge $\iff \alpha > 1$

$(\alpha = \beta - 1)$

6.9.2022

$$\sum_{n=1}^{\infty} \underbrace{n^\alpha \left[1+\sin\left(\frac{1}{n}\right) - \sqrt{1+\frac{2}{n}}\right]}_{= a_n}$$

Suggerimento: moltiplicare e dividere con

$$1+\sin\left(\frac{1}{n}\right) + \sqrt{1+\frac{2}{n}}$$

Quindi

$$a_n = n^\alpha \frac{\left(1+\sin\left(\frac{1}{n}\right)\right)^2 - \left(1+\frac{2}{n}\right)}{1+\sin\left(\frac{1}{n}\right) + \sqrt{1+\frac{2}{n}}} \qquad 1+\sin\left(\frac{1}{n}\right) + \sqrt{1+\frac{2}{n}} \sim 2 \quad n\to\infty$$

Taylor: $\sin\left(\frac{1}{n}\right) = \frac{1}{n} - \frac{1}{6n^3} + o(n^{-3}) \quad n\to\infty$

$$\Rightarrow \left(1+\sin\left(\frac{1}{n}\right)\right)^2 - \left(1+\frac{2}{n}\right)$$

$$= \left(1+\frac{1}{n} - \frac{1}{6n^3}\right)^2 - \left(1+\frac{2}{n}\right) + o(n^{-3})$$

$$= \left(1+\frac{1}{n}\right)^2 + o(n^{-2}) - 1 - \frac{2}{n}$$

$$= \frac{1}{n^2} + o(n^{-2})$$

Quindi

$$a_n \sim n^\alpha \cdot \frac{1}{2n^2} \qquad n\to\infty$$

$$= \frac{1}{2} n^{\alpha-2} \qquad n\to\infty$$

Per il criterio del confronto asintotico la serie converge se e solo se $\alpha - 2 < -1$

$\alpha < 1$

31.8.2021

$$\sum_{n=1}^{\infty} \underbrace{n^{\beta} \frac{\arctan\left(\frac{1}{n}\right) - \sin\left(\frac{1}{n}\right)}{\log(1+n)}}_{= a_n}$$

Per quali $\beta \in \mathbb{R}$ la serie converge assolutamente?

Taylor: $\sin\left(\frac{1}{n}\right) = \frac{1}{n} - \frac{1}{6n^3} + o(n^{-3}) \quad n \to \infty$

$$\arctan(x) = \underbrace{\arctan^{(1)}(0)}_{=1} \cdot x + \frac{1}{2} \underbrace{\arctan^{(2)}(0)}_{=0} x^2 + \frac{1}{6} \arctan^{(3)}(0)\, x^3 + o(x^3)$$

$$(\arctan x)' = \frac{1}{1+x^2} \Rightarrow$$

$$(\arctan x)'' = \left(\frac{1}{1+x^2}\right)' = -\frac{2x}{(1+x^2)^2}$$

$$(\arctan x)''' = -\frac{2}{(1+x^2)^2} + \frac{2 \cdot 2x \cdot 2x}{(1+x^2)^3}$$

$$\Rightarrow \arctan^{(3)}(0) = -2$$

$$\Rightarrow \arctan(x) = x - \frac{x^3}{3} + o(x^3) \quad x \to 0$$

$$\Rightarrow \arctan\left(\frac{1}{n}\right) = \frac{1}{n} - \frac{1}{3n^3} + o(n^{-3}) \quad n \to \infty$$

$$\Rightarrow \arctan\left(\frac{1}{n}\right) - \sin\left(\frac{1}{n}\right) = -\frac{1}{3n^3} + \frac{1}{6n^3} + o(n^{-3})$$

$$= -\frac{1}{6n^3} + o(n^{-3}) \quad n \to \infty$$

Quindi

$$|a_n| \sim \frac{1}{6} \frac{n^{\beta-3}}{\log n}$$

$\Rightarrow$ la serie converge assolutamente se e solo se converge la serie

$$\sum_{n=2}^{\infty} \frac{n^{\beta-3}}{\log n} \iff \beta - 3 < -1$$

$$\iff \beta < 2$$